

A Quantum Model of Schwarzschild Black Hole Evaporation*

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Abstract

We construct a one-loop effective metric describing the evaporation phase of a Schwarzschild black hole in a spherically symmetric null-dust model. This is achieved by quantising the Vaidya solution and by choosing a time dependent quantum state. This state describes a black hole which is initially in thermal equilibrium and then the equilibrium is switched off, so that the black hole starts to evaporate, shrinking to a zero radius in a finite proper time. The naked singularity appears, and the Hawking flux diverges at the end-point. However, a static metric can be imposed in the future of the end-point. Although this end-state metric cannot be determined within our construction, we show that it cannot be a flat metric.

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1 Introduction

The two-dimensional dilaton gravity models turned out to be very useful toy models of black hole formation and evaporation [1]. Their relevance for 4d black holes comes from the fact that the spherically symmetric scalar field collapse can be described by a 2d dilaton gravity action

$$S = \frac{1}{2} \int dr dt \sqrt{-g} e^{-2\phi} \left[\left(R + 2(\nabla\phi)^2 + 2e^{2\phi} \right) - \frac{1}{2} G \sum_{i=1}^N (\nabla f_i)^2 \right], \quad (1.1)$$

where G is the Newton constant and the 4d line element ds_4 is related to the 2d line element ds by

$$ds_4^2 = ds^2 + e^{-2\phi} d^2. \quad (1.2)$$

R is the 2d scalar curvature associated with the 2d metric $g_{\mu\nu}$, ϕ is the dilaton field and f_i are N matter scalar fields. These fields depend only on time t and radial coordinate r , while the angular dependence resides in d^2 . The spherically symmetric collapse was studied by several authors [2], and the problem of determining a semiclassical metric which includes the back-reaction of the Hawking radiation is still unsolved. This is related to the fact that the classical equations of motion are not solvable. In contrast to this, a string theory inspired 2d dilaton gravity model [3]

$$S_0 = \frac{1}{2} \int d^2x \sqrt{-g} \left[e^{-2\phi} \left(R + 4(\nabla\phi)^2 + 4\lambda^2 \right) - \frac{1}{2} \sum_{i=1}^N (\nabla f_i)^2 \right], \quad (1.3)$$

is classically solvable, and its solution describes a formation of a 2d black hole. The quantization of (1.3) is made simpler by the fact that the matter fields propagate freely [6, 7, 8, 9, 10, 11, 12, 13], so that the one-loop [14, 15, 10], and the two-loop [16] effective metrics were obtained. Therefore one can study analytically the back-reaction effects in this 2d model.

In this paper we are going to study a more realistic 2d dilaton gravity model, which will have some of the nice features of (1.3) but it is going to describe a 4d black hole. We will study

$$S = \frac{1}{2} \int d^2x \sqrt{-g} \left[e^{-2\phi} \left(R + 2(\nabla\phi)^2 + 2e^{2\phi} \right) - \frac{G}{2} \sum_{i=1}^N (\nabla f_i)^2 \right], \quad (1.4)$$

whose 4d interpretation is that of a self-gravitating spherically symmetric null-dust cloud. When compared to the action (1.1), one notices that a

simplifying feature of (1.4) is that the matter fields do not couple to the dilaton, so that one obtains free-field matter equations of motion in the conformal gauge, in analogy to the action (1.3). This means that the task of determining the back-reaction in the model (1.4) is going to be simpler than in the model (1.1). Still, the quantization of (1.4) is complicated by the fact that the general solution of the equations of motion is not known. However, in a special case when

$$ds^2 = - \left(1 - \frac{2m(v)}{r} \right) dv^2 + 2dvdr , \quad (1.5)$$

where $\exp(-\phi) = r$, $G = 1$, the classical solution for $f_i = f_i(v)$ is given by

$$\frac{dm(v)}{dv} = T_{vv}(v) = \frac{1}{2} \sum_{i=1}^N \left(\frac{df_i}{dv} \right)^2 , \quad (1.6)$$

which is the well-known Vaidya solution [17]. It describes a collapse of a spherically symmetric null-dust cloud. The equations (1.5) and (1.6) will be the starting point for our quantization procedure, from which we will determine an effective metric describing the one-loop back-reaction effects. Note that the back-reaction effects in the model (1.4) have been studied in [4, 5] where the one-loop back-reaction has been modeled by adding a Polyakov-Liouville term to the action (1.4). However, the resulting equations of motion are not solvable, and only a numerical study has been done.

In this paper we will perform an operator quantization of the equations (1.5) and (1.6), so that an explicit expression for a one-loop effective metric will be obtained. This metric will describe the evaporation of a Schwarzschild black hole which was initially in a thermal equilibrium state. This is achieved by using a quantization formalism developed in [10, 11], and by using the idea of thermal bath removal [18], which was developed in the case of the 2d model (1.3). We first show that the idea of thermal bath removal can be naturally formulated in the operator formalism, where it corresponds to the introduction of a time dependence in the Heisenberg quantum state of the system. This time dependence can be attributed to the external forces which switch off the thermal equilibrium. Then we implement this idea to the model (1.4) and obtain a one-loop metric whose properties we study.

2 Operator formalism and thermal bath removal in the CGHS model

The general solution of the classical equations of motion following from (1.2) in the conformal gauge $ds^2 = -e^{2\rho}dx^+dx^-$ are (up to constant shifts in the x^\pm coordinates)

$$e^{-2\rho} = e^{-2\phi} = -\lambda^2 x^+ x^- - F_+ - F_- + \frac{M}{\lambda}, \quad (2.1)$$

$$f_i = f_{i+}(x^+) + f_{i-}(x^-), \quad (2.2)$$

where

$$F_\pm = \int^{x^\pm} \int^{x^\pm} T_{\pm\pm}(x^\pm), \quad (2.3)$$

and $T_{\pm\pm} = \frac{1}{2}(\partial_\pm f)^2$. M/λ is an integration constant, and the residual conformal invariance has been fixed by the so-called "Kruskal" gauge $\rho = \phi$.

From the expressions (2.1), (2.2) and (2.3) it is clear that the independent degrees of freedom are those of the matter fields. Therefore a reduced phase space (RPS) quantization should give a physical Hilbert space which coincides with the Hilbert space of massless scalar fields propagating on a flat background [7, 10]. As far as the problem of diffeomorphism anomalies is concerned, it is formally avoided in the RPS quantization, although it may be hidden in the non-covariant form of the gauge-fixed theory. However, an anomaly-free Dirac quantization of the CGHS model gives the same physical Hilbert space as the RPS quantization [13], which guarantees the diffeomorphism invariance of the RPS results. The dynamics is generated by the free-field hamiltonian of N massless scalar fields, and therefore the quantum evolution is unitary. The effective metric is determined by $ds^2 = -\langle e^{2\rho} \rangle dx^+ dx^-$. The effective conformal factor can be evaluated perturbatively by using a matter-loop expansion [10, 11], so that at the one-loop order one obtains

$$e^{-2\rho_1} = \langle \psi_0 | e^{-2\rho} | \psi_0 \rangle = -\lambda^2 x^+ x^- - \langle F_+ \rangle - \langle F_- \rangle + \frac{M}{\lambda}, \quad (2.4)$$

where now F_\pm are operator valued expressions (2.3) in the Heisenberg picture. The initial state $|\psi_0\rangle$ can be chosen to be a coherent state $e^{A_+} |0_{\sigma^+}\rangle \otimes$

$|0_{\sigma-}\rangle$, corresponding to a left-moving pulse of matter, where σ^\pm are asymptotically flat dilaton vacuum coordinates ($\lambda x^\pm = \pm e^{\pm\lambda\sigma^\pm}$) and $|0_{\sigma+}\rangle \otimes |0_{\sigma-}\rangle$ is the corresponding vacuum. If the normal ordering in T_\pm is chosen to be with respect to the Kruskal vacuum $|0_{x+}\rangle \otimes |0_{x-}\rangle$, then [11]

$$\langle\psi_0|T_{++}|\psi_0\rangle = -\frac{N}{48\pi(x^+)^2} + \frac{1}{2}(\partial_+f)^2 \quad , \quad \langle\psi_0|T_{--}|\psi_0\rangle = -\frac{N}{48\pi(x^+)^2} \quad , \quad (2.5)$$

where we have chosen the conventional normalization of the flux [19], so that there is no a factor of $1/\pi$ in (1.3). The expression (2.4) then gives an evaporating black hole solution corresponding to the one-loop effective action of [15]

$$S = S_0 - \frac{N}{96\pi} \int d^2x \sqrt{-g} R \square^{-1} R - \frac{N}{24\pi} \int d^2x \sqrt{-g} (R\phi - (\nabla\phi)^2) \quad . \quad (2.6)$$

Note that one can consider a different process of black hole evaporation, if a different initial state is chosen. Instead of the boundary conditions (2.5), which correspond to the gravitational collapse situation, one can consider an evaporation process where initially one has a black hole in thermal equilibrium and at $x^+ = x_0^+$ the incoming thermal flux is switched off [18]. This process can be described by the boundary conditions

$$\langle T_{++} \rangle = -\frac{N}{48\pi(x^+)^2} \theta(x^+ - x_0^+) \quad , \quad \langle T_{--} \rangle = 0 \quad . \quad (2.7)$$

It is not difficult to see that the boundary conditions (2.7) correspond to the following state $|_0\rangle$

$$|_0\rangle = \theta(x_0^+ - x^+) |0_{x+}\rangle \otimes |0_{x-}\rangle + \theta(x^+ - x_0^+) |0_{\sigma+}\rangle \otimes |0_{\sigma-}\rangle \quad . \quad (2.8)$$

A novel feature of (2.8) is that the Heisenberg state $|_0\rangle$ now depends on time, which reflects the nature of the new process where an external force has to be used in order to do the switching. It follows from (2.8) that

$$\langle T_{\sigma^+\sigma^+} \rangle = \frac{N\lambda^2}{48\pi} \theta(x_0^+ - x^+) \quad , \quad \langle T_{\sigma^-\sigma^-} \rangle = \frac{N\lambda^2}{48\pi} \quad , \quad (2.9)$$

where the normal ordering in $T_{\sigma^\pm\sigma^\pm}$ is with respect to the dilaton vacuum, so that the incoming constant thermal flux has been switched off.

The one-loop solution for $x^+ > x_0^+$ is now given by

$$e^{-2\phi} = -\lambda^2 x^+ (x^- +) - \frac{N}{48\pi} \log\left(\frac{x^+}{x_0^+} + 1\right) + \frac{M}{\lambda} \quad , \quad (2.10)$$

where $\lambda = -N/48\pi\lambda^2 x_0^+$. The asymptotically flat coordinates $\tilde{\sigma}^\pm$ at the future null-infinity are given by

$$\tilde{\sigma}^+ = \sigma^+ \quad , \quad e^{-\lambda\tilde{\sigma}^-} = e^{-\lambda\sigma^-} - \lambda \quad . \quad (2.11)$$

The first relation in (2.11) is consistent with the choice (2.8), since it implies that for $x^+ > x_0^+$ there is no incoming flux at the past null-infinity, i.e.

$$\langle T_{\tilde{\sigma}^+ \tilde{\sigma}^+} \rangle = 0 \quad .$$

The second relation in (2.11) implies that the Hawking flux is the same as the initial thermal flux, i.e.

$$\langle T_{\tilde{\sigma}^- \tilde{\sigma}^-} \rangle = \frac{N\lambda^2}{48\pi} \quad . \quad (2.12)$$

It is not difficult to see that, due to the Hawking radiation, the apparent horizon shrinks and meets the curvature singularity in a finite proper time. The evaporating solution can be continuously matched to a static solution on the null line $x^- = x_{int}^-$. This solution coincides with the remnant geometry of [15], which appears in the evaporation process initiated by a gravitational collapse. Therefore this 2d example confirms the intuition that the basic features of the evaporation process do not depend on the way how the black hole was created.

3 One-loop analytic model for Schwarzschild black hole evaporation

Now we apply the idea of thermal bath removal to the model (1.4). The main problem which appears when trying to apply the RPS operator formalism to the theory (1.4) is that, in contrast to the theory (1.3), we do not know the general classical solution for an arbitrary matter energy-momentum tensor $T_{\mu\nu}$. However, if we want to describe the evaporation process of a black hole which is initially in thermal equilibrium and then the incoming thermal flux is switched off, the problem becomes simpler.

We start from the Vaidya solution (1.5), and in analogy with the 2d case, we take the following state $|0\rangle$

$$|0\rangle = \theta(v_0 - v) |0_V\rangle \otimes |0_U\rangle + \theta(v - v_0) |0_v\rangle \otimes |0_U\rangle \quad , \quad (3.1)$$

where $V = 4M \exp(v/4M)$ and $U = -4M \exp(-u/4M)$ are the Kruskal coordinates of the initial Schwarzschild black hole and M is its mass. Consistency then requires that for $v < v_0$

$$\langle T_{\mu\nu} \rangle = 0 \quad , \quad (3.2)$$

which is satisfied if $T_{\mu\nu}$ is normal ordered with respect to the Hartle-Hawking vacuum $|0_V\rangle \otimes |0_U\rangle$. The incoming and the outgoing flux are constant for $v < v_0$, and take the value corresponding to the temperature $T = (8\pi M)^{-1}$

$$\langle \tilde{T}_{vv} \rangle = \langle \tilde{T}_{uu} \rangle = \frac{N}{48\pi(4M)^2} \quad , \quad (3.3)$$

where $\tilde{T}_{\mu\nu}$ denotes the operator obtained by normal ordering $T_{\mu\nu}$ with respect to the asymptotically flat coordinates (u, v) . For $v > v_0$, one obtains

$$\langle T_{vv} \rangle = -\frac{N}{48\pi(4M)^2} = -\beta/2 \quad , \quad \langle T_{uu} \rangle = 0 \quad , \quad (3.4)$$

where the first equation follows from

$$\begin{aligned} \langle T_{vv} \rangle &= \langle 0_v | : T_{vv} :_V | 0_v \rangle = \left(\frac{dV}{dv} \right)^2 \langle 0_v | : T_{VV} :_V | 0_v \rangle \\ &= -\frac{N}{24\pi} \left(\frac{dV}{dv} \right)^2 D_V(v) \quad , \end{aligned} \quad (3.5)$$

and $D_V(v) = \frac{v'''}{v'} - \frac{3}{2} \left(\frac{v''}{v'} \right)^2$ is the Schwartzian derivative. The effective metric is then obtained by taking the expectation value of the expression (1.5), so that

$$\langle ds^2 \rangle = - \left(1 + \frac{1}{r} [\beta v \theta(v) - r_s] \right) dv^2 + 2dvdr \quad , \quad (3.6)$$

where we have used (3.2), (3.4) and (1.6). $r_s = 2M$ is the Schwarzschild radius and we have set $v_0 = 0$. The effective metric (3.6) is of the one-loop order since it is only a function of $\langle T_{\mu\nu} \rangle$ and it does not depend on $\langle T_{\mu\nu} T_{\rho\sigma} \rangle$ or on the higher-order energy-momentum tensors correlation functions.

For $v > 0$ the metric (3.6) represents an evaporating black hole whose mass is linearly decreasing with time. Such a metric was previously studied in [20], where it was ad hoc postulated and used to describe the evaporation phase of a black hole which was created from a vacuum. Consequently, a flat

spacetime was chosen for $v < 0$, instead of the Schwarzschild spacetime. The advantage of our approach is that the operator formalism provides metrics which are consistent with the boundary conditions. In this way one avoids inconsistencies which may appear due to the ad hoc nature of the procedure used in [20]. For example, a flat metric was chosen for $v > r_s/\beta$ in [20], and since the Hawking radiation is produced, one obtains a flat spacetime with non-zero energy-momentum tensor.

The line element (3.6) can be written in the conformal form (we will omit the expectation value)

$$ds^2 = -\theta(-v) \left(1 - \frac{r_s}{r}\right) dv du + \theta(v) \frac{r}{z^2 r_s} \left(1 + z - 2\beta z^2\right) dv d\tilde{u}, \quad (3.7)$$

where

$$z = \frac{r}{\beta v - r_s}, \quad (3.8)$$

and the coordinate \tilde{u} is determined from the equation

$$|1 - \beta v/r_s|^{1/\beta} e^{\tilde{u}/r_s} = |z - z_-|^{-A_-} |1 - z/z_+|^{-A_+}. \quad (3.9)$$

The constants z_{\pm}, A_{\pm} are given by

$$z_{\pm} = \frac{1 \pm \sqrt{1 + 8\beta}}{4\beta}, \quad A_{\pm} = \frac{1}{2\beta} \left(1 \pm \frac{1}{\sqrt{1 + 8\beta}}\right). \quad (3.10)$$

Note that (3.9) can be also viewed as the equation determining $r = r(\tilde{u}, v)$.

The requirement that the conformal factor in (3.7) is continuous at $v = 0$ gives

$$\frac{\tilde{u}}{r_s} = -A_- \log \left| \frac{r}{r_s} + z_- \right| - A_+ \log \left| 1 + \frac{r}{r_s z_+} \right|, \quad (3.11)$$

where

$$r + r_s \log \left| \frac{r}{r_s} - 1 \right| = -\frac{u}{2}. \quad (3.12)$$

The relations (3.11) and (3.12) determine the function $u = u(\tilde{u})$. One can now check that the incoming thermal flux has been removed for $v > 0$, since v remains the asymptotically flat coordinate at the past null infinity $\tilde{u} \rightarrow -\infty$.

The scalar curvature of the effective metric is given by

$$R = \frac{2}{r^3} (r_s - \beta v \theta(v)), \quad (3.13)$$

so that the curvature singularity is at $r = 0$. The apparent horizon curve is determined by $\partial_v r = 0$, which gives

$$r_{AH} = r_s - \beta v \theta(v) . \quad (3.14)$$

r_{AH} decreases as the black hole evaporates, and the curve (3.14) intersects the $r = 0$ curve at

$$v_{int} = r_s / \beta \quad , \quad \tilde{u}_{int} = \infty \quad , \quad (3.15)$$

so that for $v > v_{int}$ a naked singularity appears, see Fig. 1.

Since the metric (3.6) is a one-loop approximation, it is valid only in the region where $Rl_P^2 < 1$ (since the loop-expansion of the effective metric is in $\langle T^n \rangle$, which is of the order of $(Rl_P^2)^n$). This gives that $r_{AH} > \sqrt{2}l_P$, which is the expected Planck length cutoff. Note that if one defines a dynamical black hole mass M_E as $M_E = \frac{1}{2}r_{AH}$, then

$$\frac{dM_E}{dv} = -\beta/2 \quad , \quad (3.16)$$

which does not correspond to a thermal evaporation mass equation which is given by $\frac{dM_E}{dv} \propto -M_E^{-2}$. However, because β is very small (in physical units it is given by $\beta = \frac{N}{384\pi}(m_P/M)^2$ where m_P is the Planck mass), the difference between the non-thermal evaporation (3.16) and a thermal one will be noticed only when $r_{AH} < r_c \approx \frac{4}{5}r_s$. Therefore the evaporation is thermal for $r_{AH} > r_c \gg l_P$.

The Hawking flux \mathcal{T}_H can be calculated from the expression

$$\mathcal{T}_H = \langle 0 | T_{u_F u_F} | 0 \rangle = \langle 0_U | T_{u_F u_F} | 0_U \rangle = -\frac{N}{24\pi} D_{u_F}(U) \quad , \quad (3.17)$$

where u_F is the asymptotically flat coordinate at the future null infinity ($v = \infty$). By using (3.9) one can show that as $v \rightarrow \infty$

$$ds^2 \approx -\frac{2z_+}{A_+} |z_+ - z_-|^{-A_-/A_+} \left(\frac{\beta v}{r_s} \right)^{1-1/\beta A_+} e^{-\tilde{u}/r_s A_+} dv d\tilde{u} . \quad (3.18)$$

Hence the asymptotically flat coordinate u_F is given by

$$u_F = -A_+ r_s \left(e^{-\tilde{u}/r_s A_+} - 1 \right) . \quad (3.19)$$

By using (3.9) and the implicit relation $\tilde{u} = \tilde{u}(u)$ defined by (3.11) and (3.12) one can work out the Hawking flux from (3.17)

$$\mathcal{T}_H = \mathcal{T}_0 |x + z_-|^{-2A_-/A_+} |1 + x/z_+|^{-2} \left[1 + 4\beta(x + 1 + x^{-1}) \right]$$

$$+ 4\beta^2(x^2 + 2x + 4) - \frac{4}{A_+^2} \Big] \quad , \quad (3.20)$$

where $x = r(\tilde{u}, v = 0)/r_s = r(u, v = 0)/r_s$. The behavior of the flux is plotted in Fig. 2.

For early times ($\tilde{u} \rightarrow -\infty$ or $x \rightarrow \infty$) \mathcal{T}_H is close to the initial thermal value

$$\mathcal{T}_H \approx \mathcal{T}_0 x^{-4\beta} \approx \mathcal{T}_0 \exp\left(4\beta \log(2\beta) + 4\beta^2 \tilde{u}/r_s\right) \quad , \quad (3.21)$$

while for the late times ($\tilde{u} \rightarrow \infty$ or $x \rightarrow -z_-$) it diverges at the end-point $x = -z_- = 1 - 2\beta + O(\beta^2)$ as

$$\mathcal{T}_H \approx \mathcal{T}_0 |x + z_-|^{-4\beta} (1 + 8\beta) \quad . \quad (3.22)$$

This is an expected behaviour because of the presence of the naked singularity at the end-point, and it is related to the fact that $\frac{dM_E}{dv} \neq 0$ at the end-point [20]. However, as the end-point is approached, the higher-order loop corrections become relevant, and the one-loop approximation is expected to break down, so that the one-loop divergence could be removed by the higher-loop corrections. This actually happens in the CGHS case when the two loop corrections are taken into account [16]. Therefore one can expect that the higher-loop corrections will remove the naked singularity.

4 Conclusions

Note that our metric is a self-consistent semiclassical solution in the sense that its Einstein tensor is proportional to $\langle T_{\mu\nu} \rangle$ by construction. However, our metric does not satisfy an additional requirement that the Hawking flux is finite at the future null infinity [20]. In our case this means that the higher-order quantum corrections become important near the end-point. In the 2d case, the one-loop metric of [15] satisfies the both criteria; however, it is defined only in the weak-coupling region of the space-time, i.e. in the region where the higher-order quantum corrections can be neglected. Note that in our case the flux stays very close to the thermal classical value until very late times \tilde{u} . Therefore one could employ the BPP strategy of

removing the naked singularity by imposing a strong-coupling boundary at $\tilde{U} = \tilde{U}_b \approx 0$, where $\tilde{U} = -4M \exp(-\tilde{u}/4M)$, and then in the region $\tilde{U} > \tilde{U}_b$, $V > V_{int}$ try to impose a static metric such that it coincides with (3.7) at $\tilde{U} = \tilde{U}_b$ for $V > V_{int}$. Also note that $\tilde{U} = 0$ line is tangential to the $r = 0$ curve at $V = V_{int}$, so that one has the same situation as in the BPP case. The difference now is that the value of the Hawking flux is infinite at $\tilde{U} = 0$, which is problematic. This is avoided by putting the strong-coupling boundary at $\tilde{U} = \tilde{U}_b < 0$.

When $\tilde{u} = \tilde{u}_b$, then

$$ds^2 = -C_b(v)dv d\tilde{u} \quad ,$$

where $C_b(v) = 2 \left(\frac{\partial r}{\partial \tilde{u}} \right) |_{\tilde{u}_b}$, so that a trivial solution in the region $\tilde{U} > \tilde{U}_b$, $V > V_{int}$ is

$$ds^2 = -d\tilde{u} d\tilde{v} \quad ,$$

where $d\tilde{v} = C_b(v)dv$. However, this is not a good solution because $\tilde{u} \neq u$ and $\tilde{v} \neq v$, which means that radiation is present in the flat spacetime region $\tilde{U} > \tilde{U}_b$, $\tilde{V} > V_{int}$. This is no surprise, because one expects that the end-state geometry cannot be a flat space, but it should be an asymptotically flat quantum corrected vacuum geometry, and the corresponding one-loop effective action must contain additional counterterms to the Polyakov-Liouville counterterm, in analogy to the BPP case [15]. This quantum vacuum geometry cannot be determined within our construction. However, it is clear how our construction can be extended. One should find a more general class of classical solutions than the Vaidya solutions, and then quantize them according to our approach. These classical solutions could be obtained either approximately or by using the global symmetries of the theory [21].

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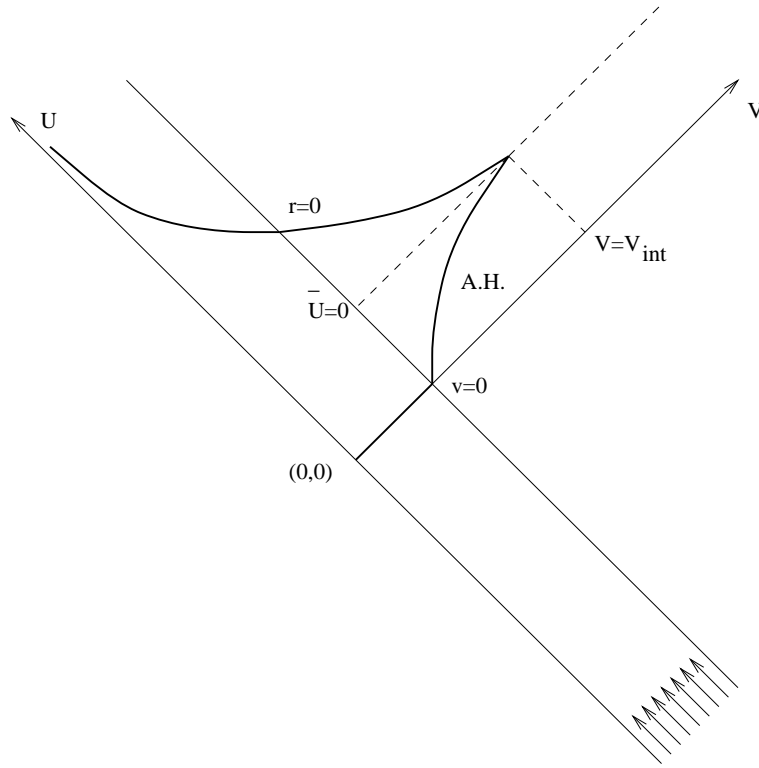


Figure I: Kruskal diagram of the one-loop geometry.

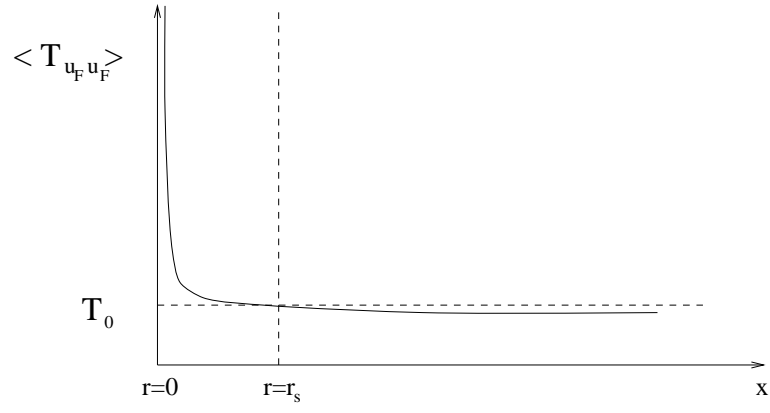


Figure II: One-loop Hawking flux.